Int. J. Mech. Sci. Vol. 31, No. 8, pp. 599–609, 1989
Printed in Great Britain.

0020–7403/89 $3.00+.00
Pergamon Press plc

SECOND ORDER INFLUENCE LINE ANALYSIS OF SUSPENSION BRIDGES

CLAUDIO FRANCIOSI and VINCENZO FRANCIOSI*
Universita' di Basilicata, Facolta' di Ingegneria, Istituto di Scienza e Tecnica delle Costruzioni, Via Nazario Sauro 85, 85100 Potenza, Italy

(*Received* 1 *August* 1988; *and in revised form* 1 *April* 1989)

Abstract—An influence line theory for suspension bridge analysis is sketched, based on a second order theory, which permits us to avoid some cumbersome procedures. The method is iterative, but the convergence is quite rapid, so allowing a preliminary check of the most dangerous load placements. Finally, a numerical example is performed, in which a recently designed one-span suspension bridge is studied.

NOTATION

B stiffening matrix
c vector of Lagrangian coordinates
c_D displacement due to $H=1$
c^*, c_s vectors of dislocations
E_c Young modulus of the cable
E_p Young modulus of the hangers
E_t Young modulus of bridge
E_t total potential energy
f sag of the cable
g dead load of the bridge
H horizontal component of the tension due to live loads
H_g horizontal component of the tension due to dead load
k concentrated stiffness
K stiffness matrix
I moment of inertia of the bridge
l central span of the bridge
l_b length of the lateral cable
l_r horizontal projection of l_b
L strain energy
M bending moments
n number of Lagrangian coordinates
P potential energy of the living loads
q_i living loads
R_B, R_C hyperstatic reactions
T shear stresses
$v^{(1)}$ first-order vertical displacements
V first-order vertical displacements matrix
α thermal coefficient
Δt thermal variation
φ single Lagrangian coordinate

1. INTRODUCTION

Usually a suspension bridge has a very large span, hence the internal forces due to moving loads are less important than the internal forces associated with earthquake, wind and thermal variations. Nevertheless, the problem should not be neglected, at least if the bridge is supposed to support heavy railway traffic. The principle of superposition is still valid, so that influence lines can be used, in order to obtain internal forces and displacements. The most obvious method is to draw these lines by letting a unit force act at a large number of sections, say n, and recording the effects of these n situations.

*This paper is dedicated by Dr C. Franciosi to Professor V. Franciosi who died recently.

This procedure is rather cumbersome, because we want to know the influence lines of internal forces and displacements at only m sections, where m is usually smaller than n, but it is important to know them as accurately as possible.

The reciprocal theorems allow us to examine the bridge in only m loading situations, in which a fictitious dislocation is supposed to act in turn at the m sections. Each of these situations leads to an influence line, which now can be calculated at a very large number, n, of sections. Basically, in the first case one is constrained to manage $n \times n$ matrices, while in the second case it is possible to use $n \times m$ matrices. Here the influence line is well approximated if n is large, while m is often very small, hence the advantage of the second method is evident.

The usual influence line theory neglects the stiffening effect of the additional horizontal load, whereas it is well known that long span suspension bridges must be calculated in a second order theory, in which this effect is taken into account [1].

In the following, a (rapidly) convergent method is presented, which allows us to use influence line theory in a second order analysis of suspension bridges. This method can be applied to the continuous structure, by following the classical deflection theory [1–11], or to the discretized structure, where the reduction can be performed within a FEM context [12–19], or in any other way [20–30]. Nevertheless, in this paper a recently proposed cell discretization method will be used, which seems to be well suited to one-dimensional structures. Moreover, suspension bridge analysis is greatly simplified if this type of reduction is first performed [31–35].

2. THE USE OF DISLOCATIONS IN THE CELL METHOD

The bridge is assumed to be discretized according to the cell method [32–33]. A dislocation in the beam is represented by n imposed values φ_i of the Lagrangian coordinates, which can be organized in the vector c_s. As an example, a continuous beam is shown in Fig. 1, in which the structure has been reduced to 13 rigid bars, connected by 14 elastic cells. The Lagrangian coordinates are the ten rotations $\varphi_1, \ldots \varphi_{10}$ of the ten bars indicated, while the redundants are the reactions R_B and R_C of the central piers.

Let this structure be subjected to dislocations, for example to an imposed rotation φ_s at the section S, and let this dislocation be denoted by D_m ($D_m = -\varphi_s$).

Firstly, D_m is allowed to act on the statically determinate system (Fig. 1b). To D_m a vector c_s corresponds, which has no associated strain energy; therefore the relative rotations between each pair of bars must be zero.

The vector c_s causes well-defined displacements $v_s(z)$. In fact, the statically determined structure has 12 Lagrangian coordinates, the ten rotations, s_B and s_C, but they are linked together by 12 conditions:

$$\begin{aligned} &\varphi_i = 0 \quad \text{if } i \neq 4, \\ &\varphi_i = -D_m \quad \text{if } i = 4, \\ &s_B = 0, \\ &s_C = 0. \end{aligned} \tag{1}$$

The displacements s_j must be removed by the hyperstatic reactions R_B and R_C. These reactions cause the displacements $v_e(z)$, which can be divided into two sets, $v_e'(z)$ and $v_e''(z)$, due respectively to the rotations c_e and to the displacements $-s_j$. The first set $v_e'(z)$ occurs on the actual structure (Fig. 1c) the second one is accompanied by zero rotations (Fig. 1d).

The solution is given by the sum of these three situations, as illustrated in Fig. 1(e). Finally, it is possible to write:

$$\begin{aligned} c &= c_s + c_e, \\ v(z) &= v_s(z) + v_e'(z) + v_e''(z). \end{aligned} \tag{2}$$

The strain energy of the system can be written as:

$$\begin{aligned} L &= \tfrac{1}{2} v_s^T R v_s + \tfrac{1}{2} v_e'^T R v_e' + \tfrac{1}{2} v_e''^T R v_e'' + v_s^T R v_e' + v_s^T R v_e'' + v_e'^T R v_e'' \\ &= L_1 + L_2 + L_3 + L_{12} + L_{13} + L_{23}. \end{aligned} \tag{3}$$

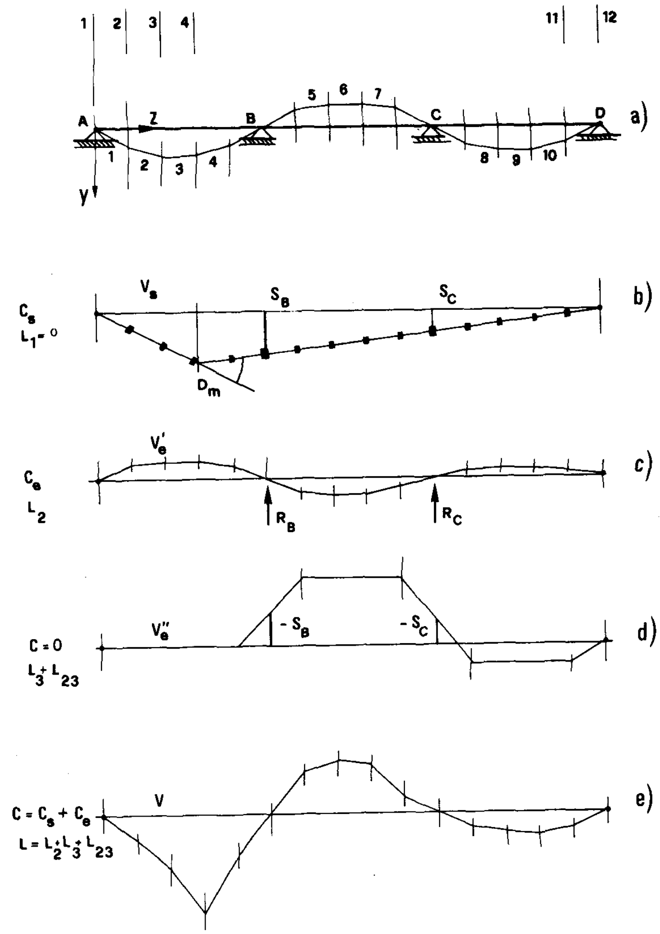

FIG. 1. The dislocation scheme in the cell discretization method.

The displacements $v_s(z)$ cause no strain energy, because everywhere $\varphi = 0$. Hence the terms L_1, L_{12} and L_{13} are zero.

The set $v_e'(z)$ causes the strain energy:

$$L_2 = \tfrac{1}{2} c_e^T K c_e, \tag{4}$$

where K is the stiffness matrix of the real structure (Fig. 1a). The set $v_e''(z)$ generate the strain energy L_3, due to the rotations of the cells; this energy is a quadratic form of the displacements s_j, and it is independent on the c_e.

Finally, the energy L_{23} exists, due to the work of the moments M_i' at the cells in Fig. 1(c) on the rotations φ_i'' in Fig. 1d. One has:

$$L_{23} = -\sum_{j=1}^{n} R_j' s_j, \tag{5}$$

where R_j' are the reactions generated by the rotations c_e on the real structure. These reactions can be expressed as a function of the Lagrangian coordinates, by means of the vectors r_j

$$R_j' = r_j^T c_e \tag{6}$$

and therefore:

$$L_{23} = -\sum_{j=1}^{n} r_j^T c_e s_j. \tag{7}$$

The potential energy P is zero, and the total energy E_t coincides with the strain energy. Therefore we have

$$E_t = \tfrac{1}{2} c_e^T K c_e - \sum_{j=1}^{n} r_j^T c_e s_j + L_2. \tag{8}$$

The equilibrium conditions give the system:

$$K c_e = \sum_{j=1}^{n} s_j r_j. \tag{9}$$

3. USE OF THE DISLOCATIONS IN SUSPENSION BRIDGE ANALYSIS

In Fig. 2 a simplified scheme of a one-span suspension bridge is given. The structure is statically redundant, and the redundancy is assumed to be the horizontal tension of the cable due to live loads H. The cable is supposed to have a parabolic shape, and its coordinates $y(z)$ are referred to the straight line EF:

$$y(z) = \frac{4f}{l^2} z(l-z). \tag{10}$$

The beam axis is also assumed to be parabolic, and its coordinates are referred to the straight line AB:

$$t(z) = \frac{4r}{l^2} z(l-z). \tag{11}$$

The n hangers divide the beam into $n+1$ bars, whose length is equal to $a = l/(n+1)$. The Lagrangian coordinates are the rotations of the first n bars. We make the same assumptions as in [31], in particular the tower is rigid, and the cable can move freely on the saddles at E and F. The horizontal component of the cable tension due to the dead load is H_g, while the horizontal component of the cable tension due to additional loads is H.

The rotations c_e of the statically determinate structure cause a horizontal displacement w_D at D equal to [31]:

$$w_D = w^T c_e, \tag{12}$$

where

$$w = \frac{8f}{(n+1)^2} \begin{bmatrix} n \\ n-1 \\ n-2 \\ \vdots \\ 1 \end{bmatrix}. \tag{13}$$

The horizontal force H acts on the structure with no rotations, because the rotations c_e are already acting with their actual value. This structure will be called a "rigid beam structure". The value of H must be given by the following congruence equation:

$$H c_D + w^T c_e = 0, \tag{14}$$

where c_D is the flexibility at D of the rigid beam structure, i.e. it is the horizontal displacement of such a structure due to $H = 1$. It is possible to write [31]:

$$c_D = 2 \frac{l_b}{l_r^2 E_c A_c} + \sum_{i=1}^{n+1} \frac{a}{E_c A_c \cos^3 \alpha_i} + \sum_{i=1}^{n+1} (\mathrm{tg}\,\alpha_i - \mathrm{tg}\,\alpha_{i-1})^2 \frac{f + c + r - y_i - t_i}{E_p A_p}, \tag{15}$$

where the meaning of all the symbols is given in the notation, and α_i is the inclination of the cable at the ith bar. The value α_i is given by

$$\mathrm{tg}\,\alpha_i = \frac{y_i - y_{i+1}}{a}. \tag{16}$$

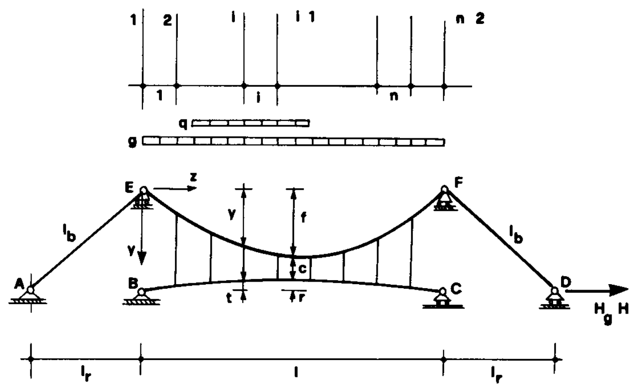

FIG. 2. The suspension bridge model.

Equation (14) furnishes:

$$H = -b^T c_e, \tag{17}$$

where $bc_D = w$.

A comparison between equations (6) and (17) shows that, for the sole unknown H:

$$r = -b \tag{18}$$

and the system (9) can be written as:

$$Kc_e = -w_D^* b, \tag{19}$$

where w_D^* is the displacement due to the imposed dislocation.

For example, a uniform thermal variation Δt causes a horizontal displacement equal to:

$$w_D^* = \alpha l^* \Delta t, \tag{20}$$

where l^* is given by:

$$l^* = 2\frac{l_b}{l_r} + \sum_{i=1}^{n+1} \frac{a}{\cos^2 \alpha_i} + \sum_{i=2}^{n+1} (\mathrm{tg}\,\alpha_i - \mathrm{tg}\,\alpha_{i-1})(f + c + r - y_i - t_i). \tag{21}$$

Every other dislocation is defined by the vector c^*, and is associated with a displacement w of the statically determinate structure given by

$$w_D^* = w^T c^* \tag{22}$$

which can be written as:

$$w_D^* = c_D b^T c^*. \tag{23}$$

The most general equilibrium condition, in the presence of applied forces q, thermal variation Δt and dislocations c^* is given by the following system:

$$Kc_e = V^T q - \alpha l^* \Delta t b - c_D (b^T c^*) b. \tag{24}$$

In this equation the matrix V allows us to calculate the displacements of the cells, according to the product:

$$v = Vc. \tag{25}$$

In the second order theory, it is necessary to take into account the second order part of the potential energy P. We have a first order term, which is due to the forces q:

$$P^{(1)} = -v^T q = -c^T V^T q \tag{26}$$

and a second order term, which is due to $H_g + H$. The second order displacement w_D is given by:

$$w_D^{(2)} = -\frac{a}{2}c^T Bc, \qquad (27)$$

where $B_{ij} = 1$ if $i \neq j$ and $B_{ii} = 2$.

It follows that the second order part of the potential energy P is equal to:

$$P^{(2)} = (H_g + H)\frac{a}{2}(c_e + c^*)^T B(c_e + c^*). \qquad (28)$$

Finally, the system (24) can be written as:

$$[K + (H_g + H)aB]c_e = V^T q - \alpha l^* \Delta t b - c_D(b^T c^*)b - (H_g + H)aBc^*. \qquad (29)$$

The additional force H can be obtained in the following way. The statically determinate structure suffers a displacement w_D given by:

$$w_D = c_D b^T c_e + \alpha l^* \Delta t + c_D b^T c^* \qquad (30)$$

and H must be able to annihilate such displacement on the rigid beam structure:

$$Hc_D + w_D = 0. \qquad (31)$$

From this condition we can easily obtain H:

$$H = -b^T c_e - b^T c^* - \frac{\alpha l^* \Delta t}{c_D}. \qquad (32)$$

4. THE RECIPROCAL THEOREMS IN THE PRESENCE OF 'MIXED' FORCES

It is well known that the reciprocal theorems (Maxwell, Betti, Volterra, etc.) can be proved even in the presence of axial forces, but these forces must remain constant. The theorems are written referring to the sole transverse loads and to their effects in the presence of the axial forces.

Sometimes we are compelled to deal with axial forces, whose corresponding displacement has also a first order part in c:

$$w_C = w_C^T c + c^T W_C c. \qquad (33)$$

In general, a force F can be classified according to the nature of the corresponding displacement s. So we have:

F is a transverse force $\leftrightarrow$ $s = s^T c$

F is an axial force $\leftrightarrow$ $s = c^T Sc$

F is a mixed force $\leftrightarrow$ $s = s^T c + c^T Sc$.

It is then possible to prove the following proposition:

Let $s_1(z)$ and $s_2(z)$ be the displacements of a structure subjected to the forces $\{G, S, F_1\}$ and $\{G, S, F_2\}$ respectively. The value G is an axial force, S is a mixed force, F_1 and F_2 are transverse forces. The reciprocal work of the set $\{F_1, S\}$ due to the displacements $s_2(z)$ is equal to the reciprocal work of the set $\{F_2, S\}$ due to the displacements $s_1(z)$.

If the first set includes also the dislocations D_1, and the second set includes also the dislocations D_2, let C_1 and C_2 be the corresponding internal forces. In this case the work of the set $\{G, S, F_1, C_1\}$ due to $s_2(z)$ and D_2 is equal to the work of the set $\{G, S, F_2, C_2\}$ due to $s_1(z)$ and D_1.

5. INFLUENCE LINES IN SECOND ORDER ANALYSIS

Let us consider the system in Fig. 3. The dislocation $D_m = -1$ is applied at the section S, and the second reciprocal theorem allows us to write

$$M_S = \int_0^l qv_1 \, dz. \qquad (34)$$

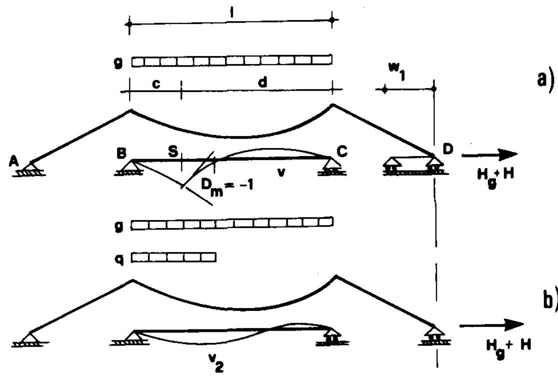

FIG. 3. A concentrated dislocation of rotational type.

If q is a uniformly distributed load, then

$$M_{S\max} = qA^{+}, \tag{35}$$

where A^{+} is the positive area of the influence line $v_1(z)$.

The structure in Fig. 3 has a roller at D, and $v_2(z)$ is calculated on the real structure. Therefore, it must be $w_{2D} = 0$, while it will be $w_{1D} = 0$. The values $v_1(z)$ and w_{1D} can be calculated with the aid of the system (29), where q and Δt are zero, and the vector c^* is given by:

$$c_i^* = \begin{cases} -\dfrac{d}{l} & \text{if} \quad i < i_S \\[2ex] \dfrac{c}{l} & \text{if} \quad i > i_S, \end{cases} \tag{36}$$

where i is the index of the generic bar, and i_S is the index of the bar to which S belongs.

The value of H should be that which is generated by the load q on the real structure (Fig. 3b). Initially we set $H = 0$, and the iterative scheme can start: the influence line is calculated on the structure in Fig. 3(a), and the load q is positioned. Then the resulting additional cable tension H due to q is calculated (Fig. 3b), and the new influence line can be detected, and so on. The iterations end when the difference between the total area of the graph $v_1(z)$ in two successive cycles is negligible. In this case we can say that H is the same in the two situations in Fig. 3, and equations (38) hold.

The strain matrix K is calculated on the real structure, in which $w_D = 0$. On the other hand, it is $w_{1D} = 0$, hence the solution $v_1(z)$ of the system (29) refers to the real structure with a dislocation equal to w_{1D}. This solution is equivalent to the solution of the statically determinate structure with the force $H_g + H$ at the roller.

If the influence line of the shear stress has to be calculated, then it is more convenient to refer to the bar, instead of the section. The dislocations c^*, in fact, have to respect the continuity of the mechanism. If t is the index of the examined bar (Fig. 4):

$$T_t = \frac{M_{t+1} - M_t}{a}. \tag{37}$$

Let the dislocation set be given by $D_m = 1$ at the left end of the bar, and by $D_m = -1$ at the right end. Therefore:

$$c_i^* = \begin{cases} \dfrac{1}{n+1} & \text{if } i \neq t \\[2ex] -\dfrac{n}{n+1} & \text{if } i = t \end{cases} \tag{38}$$

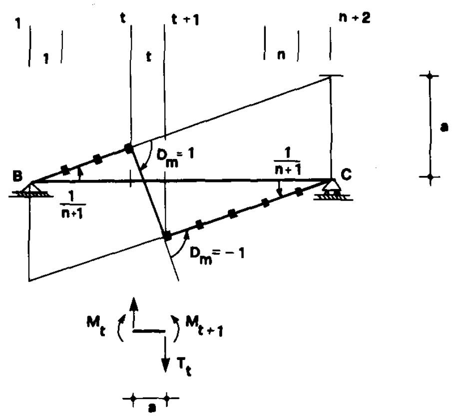

FIG. 4. A concentrated dislocation of shear type.

The second reciprocal theorem allows us to write:

$$M_{t+1}-M_t=\int_0^l qv_1\,dz \tag{39}$$

from which

$$T_t=\frac{1}{a}\int_0^l qv_1\,dz. \tag{40}$$

If q is a uniformly distributed load, then we have:

$$T_{\max}=q\frac{A^+}{a}. \tag{41}$$

If the load q acts on the structure in the presence of a thermal variation Δt, then equation (34) reads:

$$-\alpha\Delta t\sum_{i=1}^{n}N_{i1}^{(1)}l_i=\int_0^l qv_1\,dz+M_{S2}D_m \tag{42}$$

from which:

$$M_{S2}=-\frac{1}{D_m}\int_0^l qv_1\,dz-\frac{\alpha\Delta t}{D_m}\sum_{i=1}^{n}N_{i1}^{(1)}l_i, \tag{43}$$

where $N_{i1}^{(1)}$ are the first-order normal forces in the Lagrangian coordinates φ_i, and they are defined by means of the horizontal force H_1. This force is given, to a first approximation, by

$$w^Tc^*+w^Tc_e+H_1^{(1)}c_s=0 \tag{44}$$

which is equal to

$$b^Tc^*+b^Tc_e+H_1^{(1)}=0. \tag{45}$$

On the other hand:

$$b^Tc^*+b^Tc_e+H=\frac{w_D^*}{c_s}. \tag{46}$$

Therefore, it follows:

$$H_1^{(1)}=H-\frac{w_D^*}{c_s}. \tag{47}$$

Finally:

$$\sum_{i=1}^{n}N_{i1}^{(1)}=H_1^{(1)}l^* \tag{48}$$

and equation (43) becomes ($D_m = -1$):

$$M_{S2} = \int_0^l q v_1 \, dz + \alpha \Delta t H_1^{(1)} l^*. \tag{49}$$

A positive $H_1^{(1)}$ value corresponds to a negative D_m value. Hence, a positive thermal variation leads to an increase of the bending moments.

If a set of fixed forces $f(z)$ is also acting on the structure, then equation (42) becomes:

$$-\alpha \Delta t H_1^{(1)} l^* = \int_0^l f v_1 \, dz + \int_0^l q v_1 \, dz + M_{S2} D_m \tag{50}$$

or ($D_m = -1$):

$$M_{S2} = \int_0^l f v_1 \, dz + \int_0^l q v_1 \, dz + \alpha \Delta t H_1^{(1)} l^*. \tag{51}$$

Therefore, the greatest (smallest) moment is obtained by placing the load q at the positive (negative) $v_1(z)$ values:

$$M_{S2\max} = q A_1^+ + \int_0^l f v_1 \, dz + \alpha \Delta t H_1^{(1)} l^*, \tag{52}$$

$$M_{S2\min} = q A_1^- + \int_0^l f v_1 \, dz + \alpha \Delta t H_1^{(1)} l^*, \tag{53}$$

where A^+ and A^- are the positive and negative areas of the $v_1(z)$ graph.

6. NUMERICAL EXAMPLES

A recently designed suspension bridge is examined which will link Sicily to Italy. The fundamental geometrical data are given in Table 1.

The graph of the greatest and smallest bending moments is reported in Fig. 5, in which the bridge was divided into 42 rigid bars, while the moments were calculated in 24 sections.

The influence line of the bending moment at the section $z = 550$ m is sketched in Fig. 6 for three different discretization levels. In the first case the bridge was divided in 12 bars, and the

TABLE 1. GEOMETRICAL DATA OF THE MESSINA BRIDGE (COURTESY OF PROF. FINZI [36])

$l = 3300$ m
$f = 287$ m
$A_c = 5.856$ m^2
$A_p = 7.491$ m^2
$I = 6.019$ m^4
$E_t = E_p = 2.1 \times 10^6$ tm^{-2}
$E_c = 1.8 \times 10^6$ tm^{-2}
$g = 94$ tm^{-2}
$q = 20$ tm^{-2}
$\Delta t = 25$°C

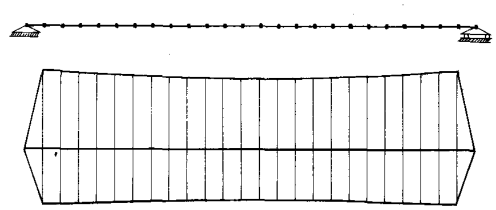

FIG. 5. Greatest bending moments of the Messina bridge.

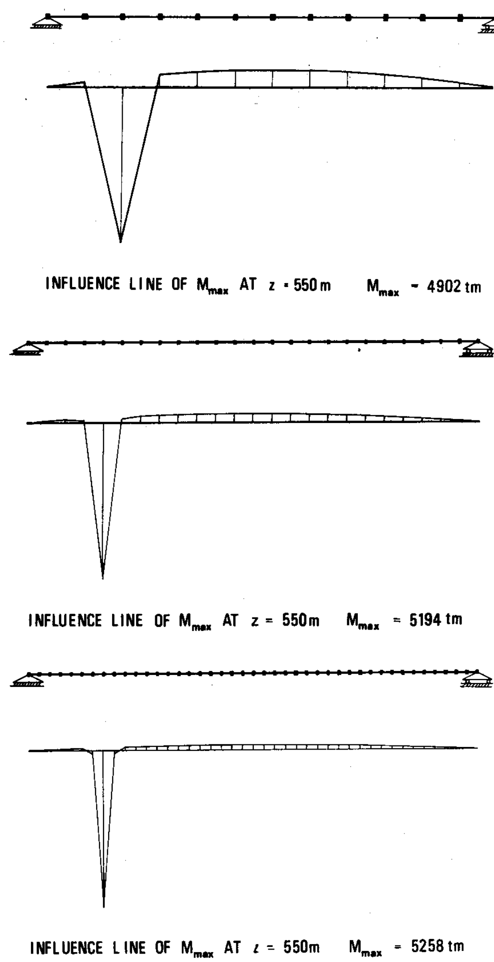

FIG. 6. Three influence lines for three different discretization steps.

moment was equal to 4902 tm, if the structure is divided in 24 bars, then the moment is equal to 5194 tm. Finally, a subdivision into 42 bars leads to a moment equal to 5258 tm. These convergence results justify the use of 42 rigid bars in the moment graph.

7. CONCLUSIONS

The influence line theory for suspension bridges has been critically reviewed and the reciprocal theorems shown to be valid if certain hypotheses can be satisfied. It is then possible to use the so-called Land method, in order to draw directly the influence lines at certain sections, by imposing a concentrated dislocation. The influence of thermal variations can also be considered. Finally, a recently designed suspension bridge has been examined, which should have the longest span of the world and the graph of the greatest bending moments has been calculated.

REFERENCES

1. D. J. PEERY, An influence-line analysis for suspension bridge. *Trans. ASCE* **121**, 463–510 (1955).
2. S. O. ASPLUND, Deflection theory analysis of suspension bridges. *IABSE Publications*, Vol. 7 (1949).
3. F. BLEICH, C. B. MCCULLOUGH, C. B. ROSECRANS and G. S. VINCENT, *The Mathematical Theory of Vibration in Suspension Bridges.* U.S. Bureau of Public Roads, Government Printing Office, Washington D.C. (1950).
4. D. B. STEINMAN, *A Practical Treatise on Suspension Bridges.* Wiley, New York (1953).

5. A. HAWRANEK and O. STEINHARDT, *Theorie und Berechnung der Stahlbrucken.* Springer, New York (1958).
6. S. A. SAAFAN, Theoretical analysis of suspension bridges. *J. Struct. Div. (ASCE)* **92**, ST4, 1–11 (1966).
7. A. G. PUGSLEY, *The Theory of Suspension Bridges.* Edward Arnold, London (1968).
8. J. SZIDAROVSKY, Corrected deflection theory of suspension bridges. *J. Struct. Div. (ASCE)* ST11, 25–51 (1960).
9. V. FRANCIOSI, Il calcolo dei ponti sospesi attraverso la teoria della "linea d'influenza del secondo ordine". Atti Ist. S.d.C. PISA n. 76 (1960).
10. H. H. WEST and A. R. ROBINSON, Continuous method of suspension bridge analysis. *J. Struct. Div. (ASCE)* **94**, ST12, 2861–2883 (1968).
11. A. JENNINGS, Gravity stiffness of classical suspension bridges. *J. Struct. Div. (ASCE)* **109**, ST1, 16–37 (1983).
12. S. G. ARZOUMANIDIS and M. P. BIENEK, Finite element analysis of suspension bridges. *Comput. Struct.* **21**, 1237–1253 (1985).
13. A. M. ABDEL-GHAFFAR, Vertical vibration analysis of suspension bridges. *J. Struct. Div. (ASCE)* **106**, ST10, 2053–2075 (1980).
14. A. M. ABDEL-GHAFFAR and L. I. RUBIN, Lateral earthquake response of suspension bridges. *J. Struct. Div. (ASCE)* **109**, ST3, 664–675 (1983).
15. A. M. ABDEL-GHAFFAR, Free lateral vibrations of suspension bridges. *J. Struct. Div. (ASCE)* **104**, ST3, 503–525 (1978).
16. A. M. ABDEL-GHAFFAR, Free torsional vibrations of suspension bridges. *J. Struct. Div. (ASCE)* **105**, ST4, 767–788 (1979).
17. A. M. ABDEL-GHAFFAR and L. I. RUBIN, Vertical seismic behaviour of suspension bridges. *Int. J. Earthquake Engng Struct. Dynamics* **11**, 1–19 (1983).
18. A. M. ABDEL-GHAFFAR and L. I. RUBIN, Suspension bridge response to multiple-support excitations. *J. Mech. Div. (ASCE)* EM2, 419–435 (1982).
19. F. VAN DER WOUDE, Natural oscillations of suspension bridges. *J. Struct. Div. (ASCE)* **108**, ST8, 1815–1830 (1982).
20. H. H. WEST and D. L. CARAMANICO, Initial value discrete suspension bridge analysis. *Int. J. Solids Struct.* **9**, 1087–1105 (1973).
21. H. H. WEST, J. E. SUHOSKI and L. F. GESCHWINDNER, Natural frequencies and modes of suspension bridges. *J. Struct. Div. (ASCE)* **110**, ST10, 2471–2487 (1984).
22. H. H. WEST, J. E. SUHOSKI and L. F. GESCHWINDNER, Natural vibrations of suspension cables. *J. Struct. Div. (ASCE)*, ST11 (1975).
23. A. S. VELETSOS and G. R. DARBRE, Free vibrations of parabolic cables. *J. Struct. Div. (ASCE)* **109**, ST2, 503–519 (1983).
24. A. S. VELETSOS and G. R. DARBRE, Dynamic stiffness of parabolic cables. *Int. J. Earthquake Engng Struct. Dynamics* **11**, 367–401 (1983).
25. H. M. IRVINE, The estimation of earthquake-generated additional tension in a suspension bridge cable. *Earthquake Engng Struct. Dynamics* **8**, 267–273 (1980).
26. H. M. IRVINE and T. K. CAUGHEY, The linear theory of free vibrations of a suspended cable. *Proc. R. Soc. (Lond.)* **A341**, 299–315 (1974).
27. H. M. IRVINE and J. H. GRIFFIN, On the dynamic response of a suspended cable. *Earth. Engng Struct. Dynamics* **4**, 389–402 (1976).
28. W. M. HENGHOLD, J. J. RUSSELL and J. D. MORGAN, Free vibrations of a cable in three dimensions. *J. Struct. Div. (ASCE)* STS (1977).
29. T. FUKUDA, Multispan suspension bridges under lateral loads. *J. Struct. Div. (ASCE)* **94**, ST1, 133–152 (1968).
30. T. FUKUDA, Analysis of longitudinally loaded suspension bridges. *J. Struct. Div. (ASCE)* **94**, ST4, 855–874 (1968).
31. C. FRANCIOSI and V. FRANCIOSI, Suspension bridge analysis using Lagrangian approach. *Comput. Struct.* **26**, 499–512 (1987).
32. C. FRANCIOSI, Three subroutines for the analysis of suspension bridge. *Comput. Struct.* **28**, 1449–153 (1988).
33. Y. YAMADA and I. KONISHI, Earthquake response and earthquake resistant design of long span suspension bridges. Proc. 3rd World Conf. Earthquake Engng New Zealand, Vol. III, IV, 312 (1965).
34. Y. YAMADA and I. KONISHI, Earthquake responses of a long span suspension bridge. Proc. 2nd World Conf. Earthquake Engng Vol. II, 863–878 (1960).
35. S. S. TEZCAN and S. CHERRY, Earthquake analysis of suspension bridges. Proc. 4th World Conf. Earthquake Engng, Santiago, Vol. II-A3, 124–140 (1969).
36. L. FINZI, NOVA, CASTELLANI, Relazione di progetto di fattibilita'—Societa' Stretto di Messina (1986).