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The timber octagon of Ely Cathedral

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The construction and basic action of the timber vault over the crossing of Ely Cathedral is examined. The structure still contains its 14th century members, but it was restored and modified in the 18th century and again in the 19th century. Stripped of its bracing, the main vault is a space truss with the property of being simultaneously a mechanism of one degree of freedom and a structure with one statical indeterminacy. This has led to some distress in the past, although the restoration of 200 years ago has provided an architectural masterpiece with a satisfactory supporting structure.

Introduction and brief architectural history

Queen Etheldreda, daughter of Anna, King of East Anglia, founded a monastery in the Isle of Ely in 673. She died six years later and was succeeded as second Abbess by her sister, Sexburga. Sexburga removed the body of Etheldreda from its grave and translated it to the monastery as an object of worship, and Sexburga herself was in due course buried behind the shrine of her sister. Sexburga's daughter Eormenilda duly became the third Abbess and was in turn enshrined, as was the body of a fourth member of the family, Wihtburga, who was Etheldreda's sister. All four were later canonized.

2. The four shrines were preserved despite the sacking of the abbey by the Danes in 870, and a new (Benedictine) monastery was consecrated in 970. In 1071 William the Conqueror took possession of the site, and he appointed a Norman kinsman, Simeon, as Abbot in 1081. Simeon determined to rebuild the whole of the monastery, and construction of the present church of St Mary, St Peter and St Etheldreda, Ely Cathedral, was started.

3. By 1106 the building, begun as usual at the east end, was so far advanced that the bodies of the four saints were all rearranged in the new presbytery, to the east of the high altar. By 1189, then, the choir (not the present choir), the crossing and transepts and the nave had all been finished, and the great West Tower[1] had been built to about two-thirds of its present height.

4. The shrines of the four saints, and other relics owned by the cathedral, attracted great numbers of pilgrims, and they were an important source of revenue for the monastery. The Norman choir could no longer accommodate the crowds, and Bishop Hugh de Northwold extended the choir by six bays between 1234 and 1251. This Gothic extension is the present termination of the cathedral.

5. The crossing was at this time surmounted by a second tower, which was part of Abbot Simeons's 12th century design; this tower collapsed on 12 February 1322.

Written discussion closes 14 October 1985; for further details see p. ii.
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6. There are records of many church towers having fallen within 20 years of completion, and it is no accident that the timescale for settlement of a crossing tower, say 12 m square, is about a generation. If during this period of consolidation the settlement is more or less uniform, then the tower may well survive for centuries. The question why the tower at Ely should stand for nearly 200 years and then collapse is probably unanswerable. There is some evidence that one of the periodic attempts at drainage of the Fens had taken place a few years earlier,[2] and this may have provoked renewed settlement. Stewart[3] talks of the 'rottenness of the tower piers' as the cause of the collapse. Whatever the reason, it is thought that the monks had had warning of the approaching ruin and had abandoned the east end of the cathedral, holding their services elsewhere. (However, the Lady Chapel, to the north east of the cathedral and alarmingly close to the crossing, had been started only a year before, in 1321.)

7. However all this may be, the original four Norman bays of the choir were rebuilt in the 1320s, and at the same time an imaginative reconstruction was made of the central crossing. This reconstruction is associated with the name of Alan de Walsingham, the then Sacrist of Ely, and he is often regarded as the architect of the present timber octagon. There is no evidence for this; he was certainly, from the point of view of the cathedral, in charge of the work. Equally certainly the best possible technical advice was obtained, and the design of the timber vault and lantern, which is the subject of this Paper, was in the hands of William Hurley, the King's Carpenter.

8. However, it may have been Walsingham who had the idea of suppressing the original crossing piers. Fig. 1 indicates the original arrangement of the piers in the region of the crossing, and Fig. 2 shows the present arrangement. The 12 m square crossing has been opened up to an octagonal space of about double that size. The foundations of the eight Norman piers were enlarged and strengthened, and the piers themselves were encased within new masonry. It may be noted that the octagon is not equilateral; there are four long and four short sides.

9. The eight piers support arches which in turn carry the octagonal masonry shell which forms the present crossing tower of the cathedral. This masonry work was completed by 1328. The octagon is vaulted in timber, Fig. 3, and the wooden ribs spring from the same capitals on the piers as the masonry arches. The timber vault itself carries an octagonal timber lantern, closed in turn by a wooden fan vault. All this work on the octagon cost £2406 (a figure to be multiplied by the order of 1000 to give a 1985 equivalent).

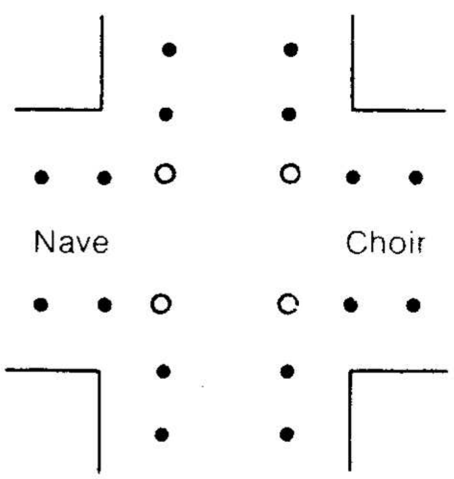

Fig. 1

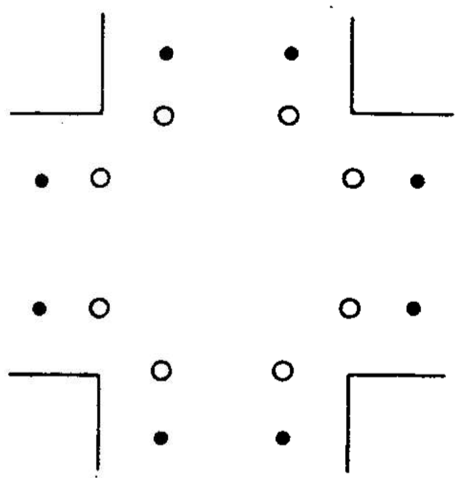

Fig. 2

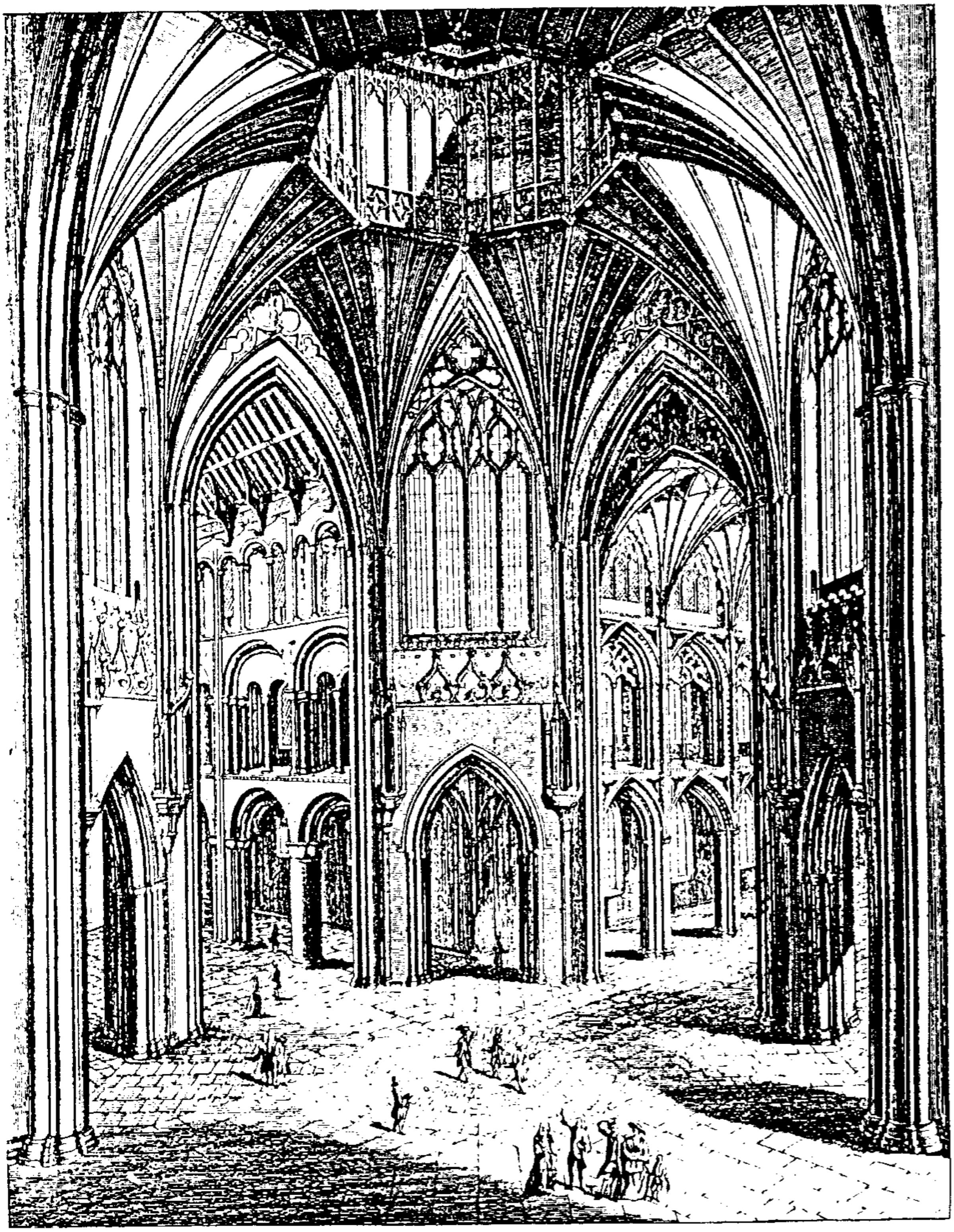

Fig. 3

10. It is of passing interest to note that the cathedral at Siena also has an enlarged crossing, of slightly smaller diameter than at Ely, However, the crossing is hexagonal, and the six piers support a masonry dome, which was completed about 1264.

11. The shrine of St Etheldreda had been preserved once more during the collapse of the tower and the construction of the octagon and was certainly in its central position at the east end of the church in the 15th century, although some of

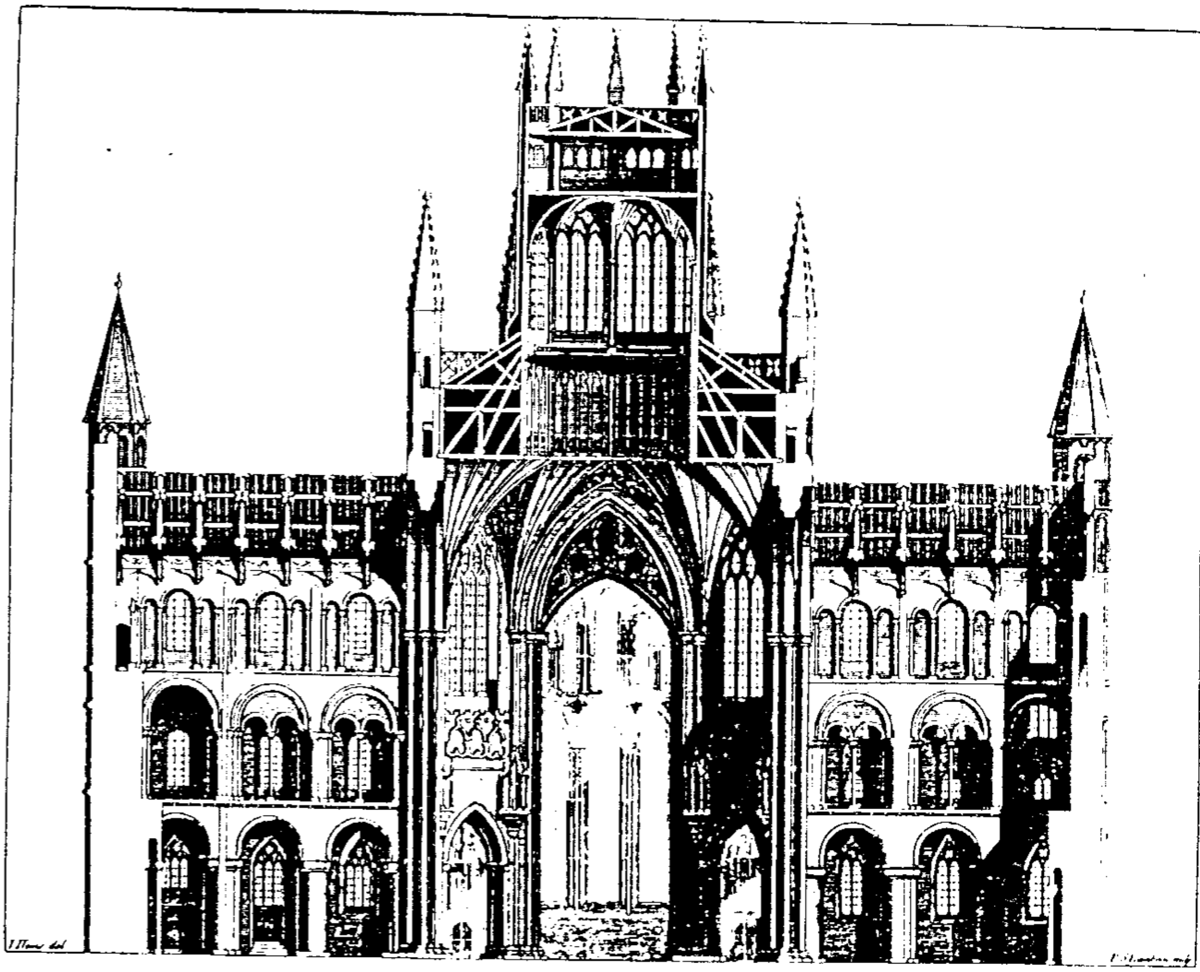

Fig. 4

the other saints had been moved. Some time after this, however, the shrine was destroyed, and the present resting place of St Etheldreda is unknown.

The basic structure

12. Figure 4 shows a sectional view of the cathedral at the crossing, looking east, from a drawing[4] engraved in 1766. The timber truss-work, partly hidden by the boards of the vaulting, is not all original, some being due to Essex in the 18th century. The capitals of the piers from which the vault springs are at about 19 m above the floor, and the vault rises to an octangular timber ring, the lower sill, at about 29 m. This lower sill is a regular octagon with side about 6 m, and it supports the eight great posts (18 m long) of the lantern proper. Fig. 5 shows an external view of the crossing, looking north, also from Bentham, but drawn in 1756. There are some considerable differences in detail between Figs 4 and 5, which will be discussed later; Essex carried out his restoration from 1757 to 1762, so that Fig. 5 is pre-Essex, and Fig. 4 shows the state after Essex had finished his work.

13. Hewitt's conjectural framing[5] of the original timber structure of the vault is shown in Fig. 6. (All this is hidden from below, being covered with boarding and non-structural ribs, Fig. 3.) The eight corner posts were erected first, set into vertical slots in the external masonry piers. These posts were then tied back against the masonry and strained apart by horizontal members, Fig. 7. From this point it is possible to add members in an orderly way to arrive at the framing of Fig. 6.

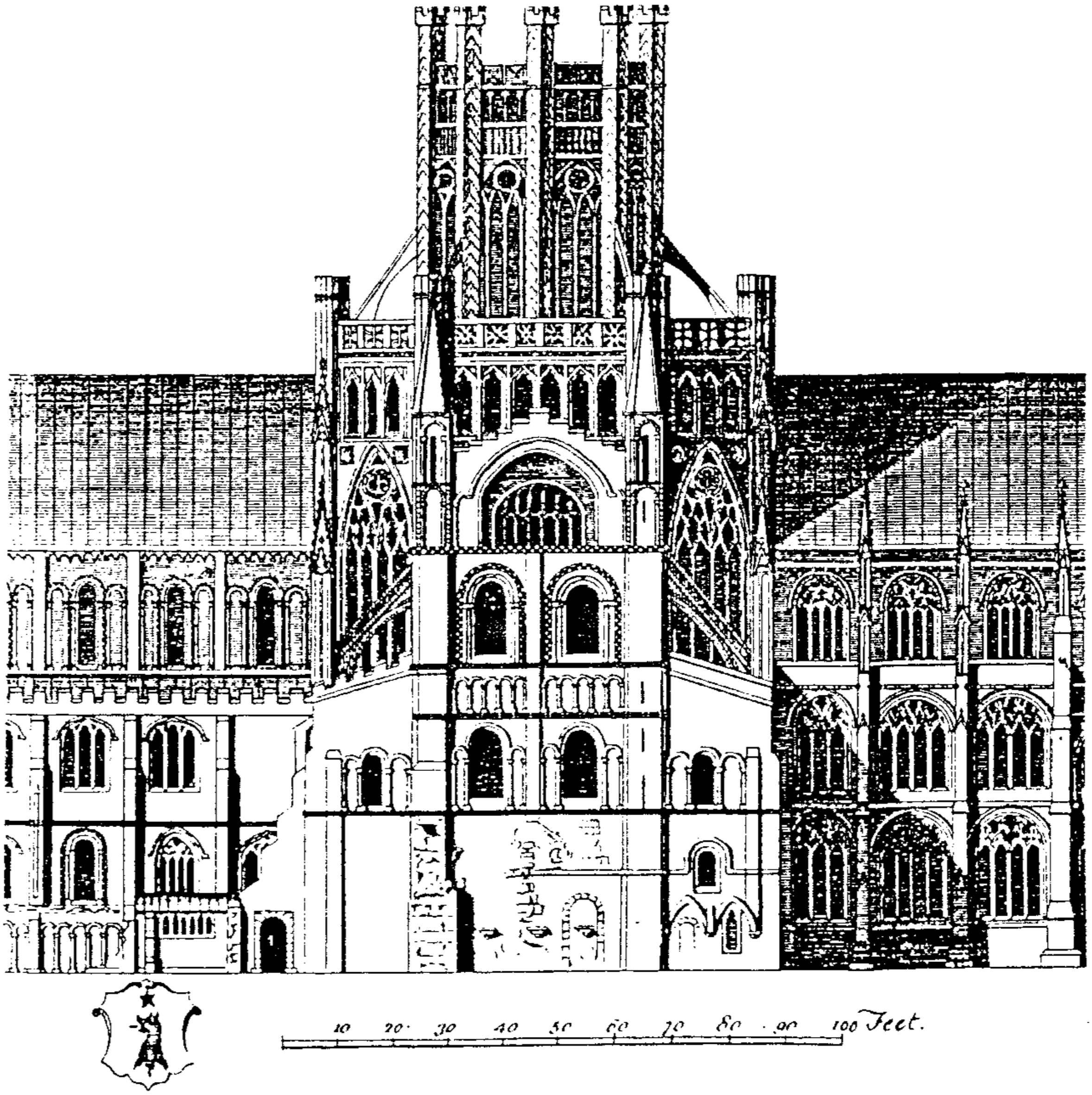

Fig. 5

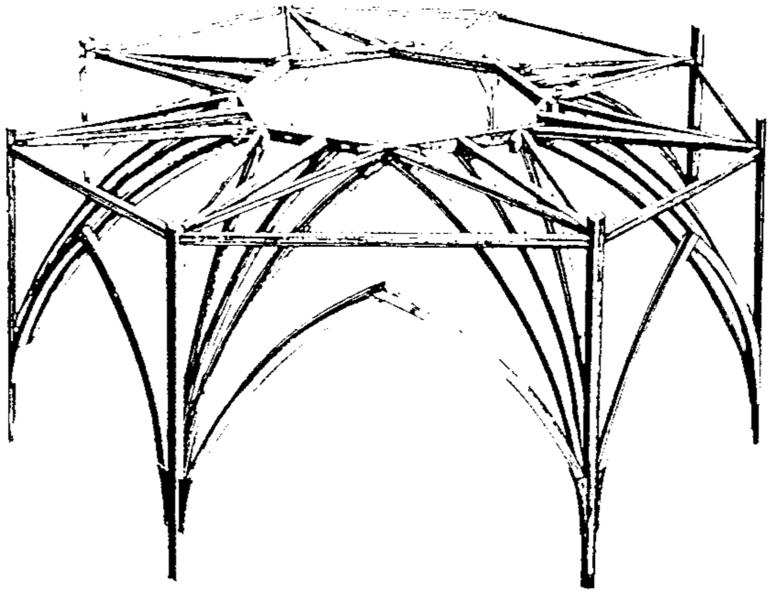

Fig. 6

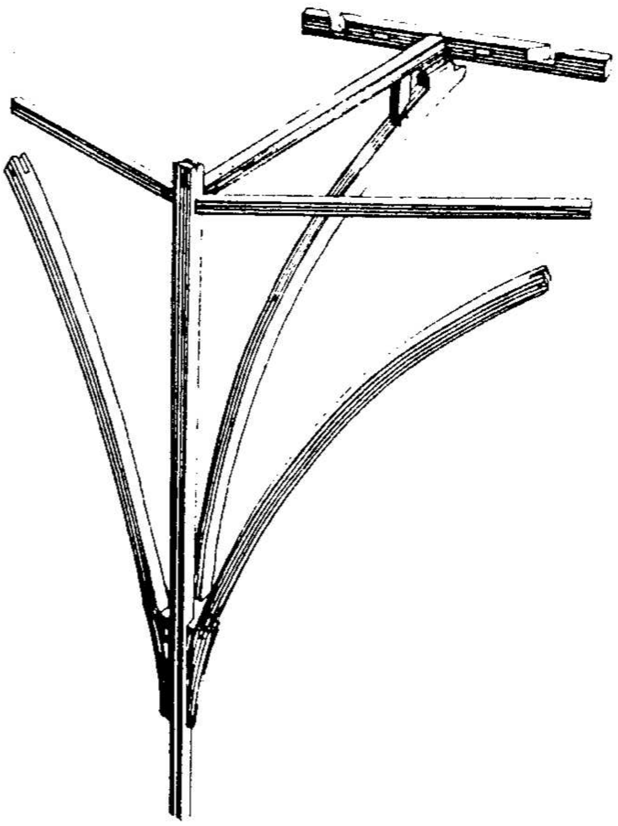

Fig. 7

14. The skeletal structure of Fig. 8, which shows the main members of the more complex real structure, is of considerable theoretical interest. The analysis of such 'reticulated domes' (Netzwerkkuppel) was treated by Föppl,[6] and discussion may be found also in Timoshenko and Young.[7] More recently, Pellegrino and Calladine[8] have made a detailed study of structures which are simultaneously both statically and kinematically indeterminate, which, as will be seen, is of present relevance.

15. If node A of the space frame of Fig. 8 is unloaded, then none of the three members meeting there can carry load, and they could be removed without in

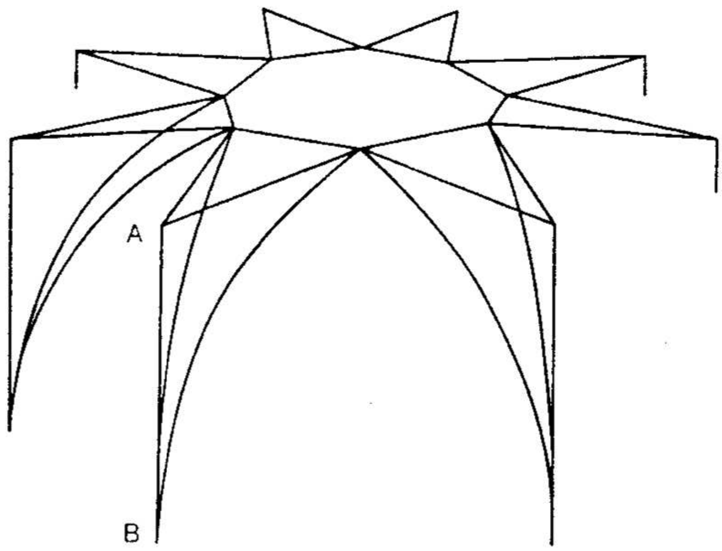

Fig. 8

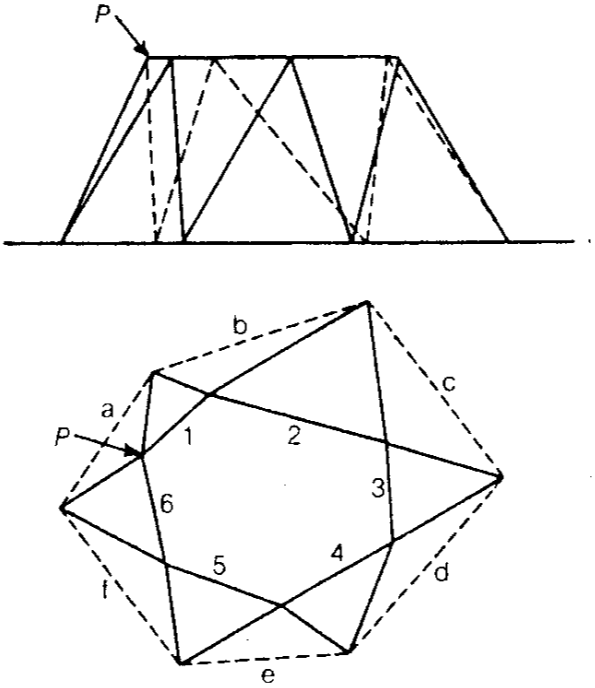

Fig. 9

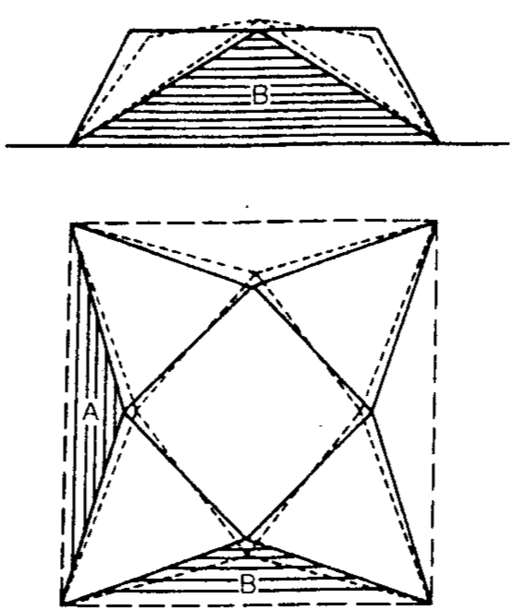

Fig. 10

theory affecting the action of the truss as a whole. In fact the vertical posts AB no longer exist at Ely, although the corresponding slots in the masonry are clearly visible; the horizontal members remain to form the floor, but not as a necessary part of the main framework.

16. Thus the essential framework at Ely is like that shown in Fig. 9, which Föppl uses to make a general study of this type of construction. The count of bars and nodes in Fig. 9 (a hexagon is shown, but the polygon may have any number of sides) indicates that the truss, viewed as a pin-jointed structure, is statically determinate. This indication is correct if the polygon has an odd number of sides.

17. If the polygon has an even number of sides, however, then the truss is once redundant and simultaneously has one degree of freedom. Calladine[9] has generalized Maxwell's count to the form

$$b - 3j = s - m \tag{1}$$

where b is the number of bars and j is the number of joints of a properly supported space framework. On the right-hand side of this equation, s is the number of statical indeterminacies and m the number of kinematical indeterminacies. Polygonal frameworks like that of Fig. 9 have $b - 3j = 0$. The curious behaviour of the even-sided polygon is illustrated for the quadrangle, again from Föppl, in Fig. 10; the quadrangle can lozenge as shown. In consequence such structures with four, six or eight sides are not good load-bearing constructions. The basic structure at Ely, Fig. 8, corresponds to an extremely flexible framework, capable in theory of carrying only very special combinations of loads, and then only in precarious equilibrium.

18. As a first step in the analysis of the timber framework of the vault, it will be assumed that the whole of the octagonal superstructure imposes eight equal loads P at the nodes of the inner ring (the lower sill) in Fig. 8. For reasons already discussed, the three members meeting at A may be removed from the analysis. The curved ribs supporting the lower sill will be scrutinized in detail later; their sup-

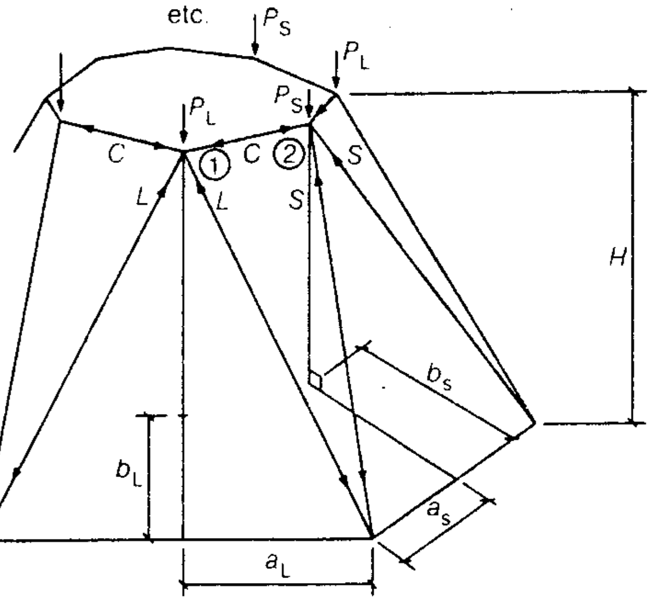

Fig. 11

porting action will be in a line joining the ends of the member, so that the problem to be solved is that of finding the forces in the basic truss of Fig. 11. It will be seen that the superstructure has *not* been represented by eight equal loads P, but by four equal loads P_L and four equal loads P_S (the suffices L and S refer to nodes of the inner ring supported respectively from the long and short sides of the outer octagon).

19. If the analysis is tackled in a straightforward way, then symmetry would indicate that the two supporting members at node 1 of Fig. 11 will carry equal forces L. Resolution of forces in a horizontal plane in a direction at right angles to the radius then shows that the two compressive forces C in the lower sill members meeting at node 1 are also equal, as indeed is again evident from notions of symmetry. Consideration of adjacent nodes shows that all eight members of the lower sill carry the same compressive load C. Two further resolutions at node 1, horizontally in the radial direction and vertically, give

$$\begin{aligned} b_L l &= C \sin 22{\cdot}5^\circ \\ 2Hl &= P_L \end{aligned} \tag{2}$$

where l is the tension coefficient of the bar force L, H is the height of the octagon and b_L is the difference in radial dimension at node 1 (i.e. measured horizontally) between the inner octagon and the supporting octagon.

20. Equations (2) solve to give

$$P_L = \frac{2HC \sin 22{\cdot}5^\circ}{b_L} \tag{3}$$

Similarly, consideration of equilibrium at node 2 leads to the equation

$$P_S = \frac{2HC \sin 22{\cdot}5^\circ}{b_S} \tag{4}$$

Now, if W is the total weight of the superstructure, then

$$4(P_S + P_L) = W \tag{5}$$

so that

$$P_L = \frac{\frac{1}{4}Wb_S}{b_L + b_S}$$
$$P_S = \frac{\frac{1}{4}Wb_L}{b_L + b_S} \quad (6)$$

21. Thus a first manifestation of the curious behaviour of the reticulated dome of Fig. 11 has become apparent. The straightforward 'engineer's' assumption that the superstructure can be replaced by eight equal loads P is not viable. The structure demands that the loads shall be unequal. For the dimensions at Ely, equations (6) indicate that $P_L = 0{\cdot}132W$ and $P_S = 0{\cdot}118W$, instead of the equal-valued $W/8$.

22. This inequality of loading is a consequence of the fact that the structure is a mechanism of one degree of freedom. Equal loads cannot be supported, and an attempt to impose them would excite the mechanism inherent in the structure, leading to large distortions. In practice, bracing members will come into action until the loads acting on the structure are divided into such proportions that equilibrium is possible (the action of some of these bracing members will be indicated later).

23. The particular equilibrium proportions of equations (6) are special values of a more general result. If the nodes of the inner ring are numbered 1, 2, . . . 8, then the general condition for equilibrium is

$$(P_1 + P_3 + P_5 + P_7)b_L - (P_2 + P_4 + P_6 + P_8)b_S = 0 \quad (7)$$

This is, of course, a work equation. The mechanism motion (of one degree of freedom) of the truss of Fig. 11 will involve (to some scale) downward movements of magnitude b_L at joints 1, 3, 5 and 7 and upward movements b_S at joints 2, 4, 6 and 8 (cf. Fig. 10), or vice versa. Thus the framework can support an infinite range of loads $P_1, P_2 \ldots P_8$ as long as equation (7) is satisfied, i.e. if seven of the loads are specified, then precarious equilibrium can be achieved, but only for a particular value of the eighth load.

24. So far, the self-stressing property of the framework has not been considered. The skeleton has been redrawn in Fig. 12 without external loads. If each node is considered in turn, it is easily found that the truss can sustain the forces shown. The members of the inner octagonal ring are all subjected to a load of the same numerical value X but are alternately in tension and in compression. (Such an alternation is possible for a ring with an even number of sides, but not with an

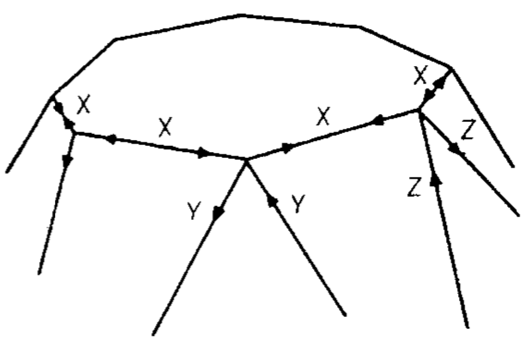

Fig. 12

odd number.) The forces in the supporting members are given by

$$a_L y = X \cos 22{\cdot}5° = a_S z \tag{8}$$

where y and z are tension coefficients, and a_L and a_S are the half-lengths of the sides of the supporting octagon (see Fig. 11). Thus all forces may be found in terms of X, but the value of X is itself unknown; the structure has a single redundancy.

25. For the symmetrical distribution of load in Fig. 11 the value of the redundancy is zero; i.e. if the forces in Figs 11 and 12 are superimposed to give a general system of equilibrium forces, then (for example) the loads in the two long supports will be $-L-Y$ and $-L+Y$. Similar pairs of expressions occur for each pair of bars in the framework, and a simple virtual work or 'strain energy' analysis then shows that X (and hence Y and Z from equations (8)) is zero.

Walsingham's (Hurley's) structure

26. The outer octagon is contained within a square of about 24 m side. Dimensions of the real structure are not uniform, and leading values used for the analysis are shown in Fig. 13.

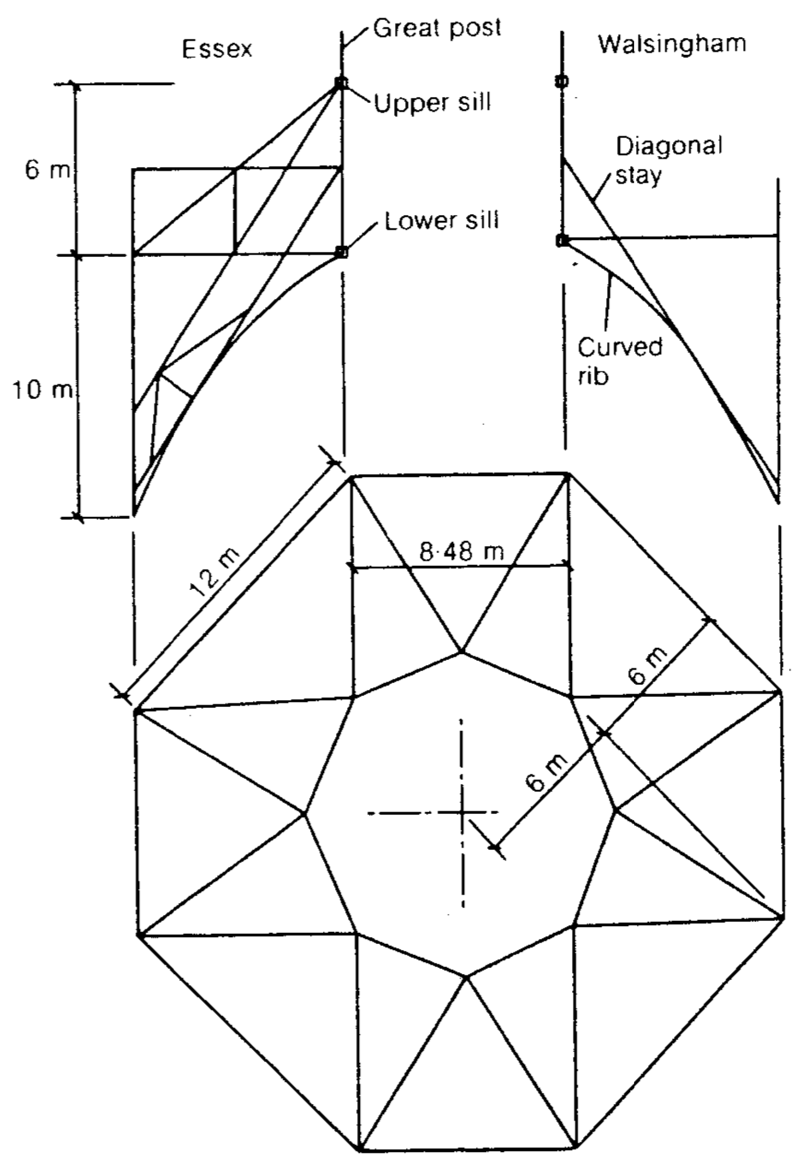

Fig. 13

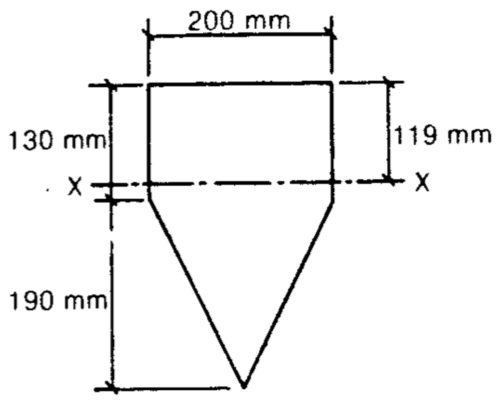

Fig. 14

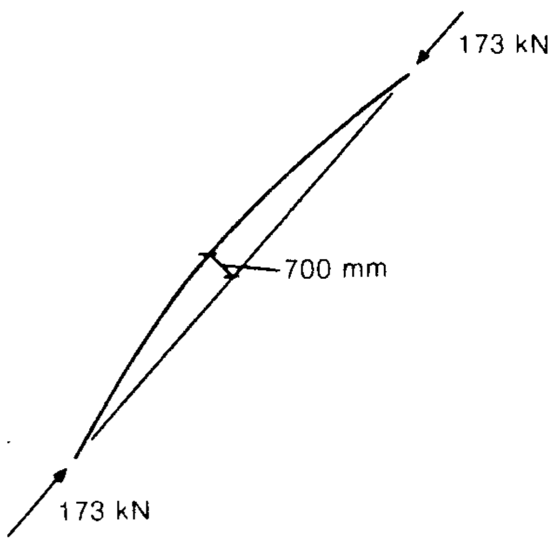

Fig. 15

27. The total weight of the lantern may be evaluated by estimating the quantities of timber, glass, lead etc. used in its construction. In round numbers, an acceptable value of the total dead load W is 2000 kN. Thus, from equations (6), the values of P_L and P_S in Fig. 11 may be found to be 264 kN and 236 kN respectively, and the value of L is then about 173 kN.

28. The cross-section of the curved rib is shown, idealized, in Fig. 14. Section properties may be estimated, and in particular the cross-sectional area is 45×10^3 mm^2, and the section modulus for bending about the axis XX has the values $1{\cdot}26 \times 10^6$ mm^3 for the compression side and $2{\cdot}12 \times 10^6$ mm^3 for the tension side. Thus the direct compressive stress under the action of 173 kN is less than 4 N/mm^2.

29. The curved rib acts as shown in Fig. 15, and at the section of maximum bow the bending stresses are 96 N/mm^2 and 57 N/mm^2 in compression and tension respectively. These values of stress are high, being of the order (and perhaps in excess) of the strength of the oak rib. It may be taken as axiomatic that a condition (necessary, but not sufficient) for the survival of a medieval structure is that the stresses should be low, at their highest an order of magnitude less than the strength of the material. Thus the direct compressive stress of 4 N/mm^2, at one-hundredth (two orders of magnitude) of the strength of the oak is satisfactory, and, if this were the only stress present, the rib would have a good margin for the acceptance of live load and for decay of the timber itself. By contrast, the calculated values of bending stress indicate that the structure cannot be acting in the way assumed. Other members of the framework must be involved in carrying the load.

30. When construction of the vaulting and floor had reached the stage shown in Fig. 6, the eight great posts could then be erected at the corners of the inner octagon. A second inner octagonal ring connected together these eight posts at a height of about 6 m above the floor (the 'upper sill' in Fig. 13), and 16 diagonal stays were introduced as shown in Hewitt's[10] drawing, Fig. 16. Other members were added, and the timber work was roofed in (cf. Fig. 4), through which the eight posts projected to form the glazed lantern lighting the crossing. A wooden fan vault was erected near the tops of these posts, and the whole lantern was closed with a final timber roof.

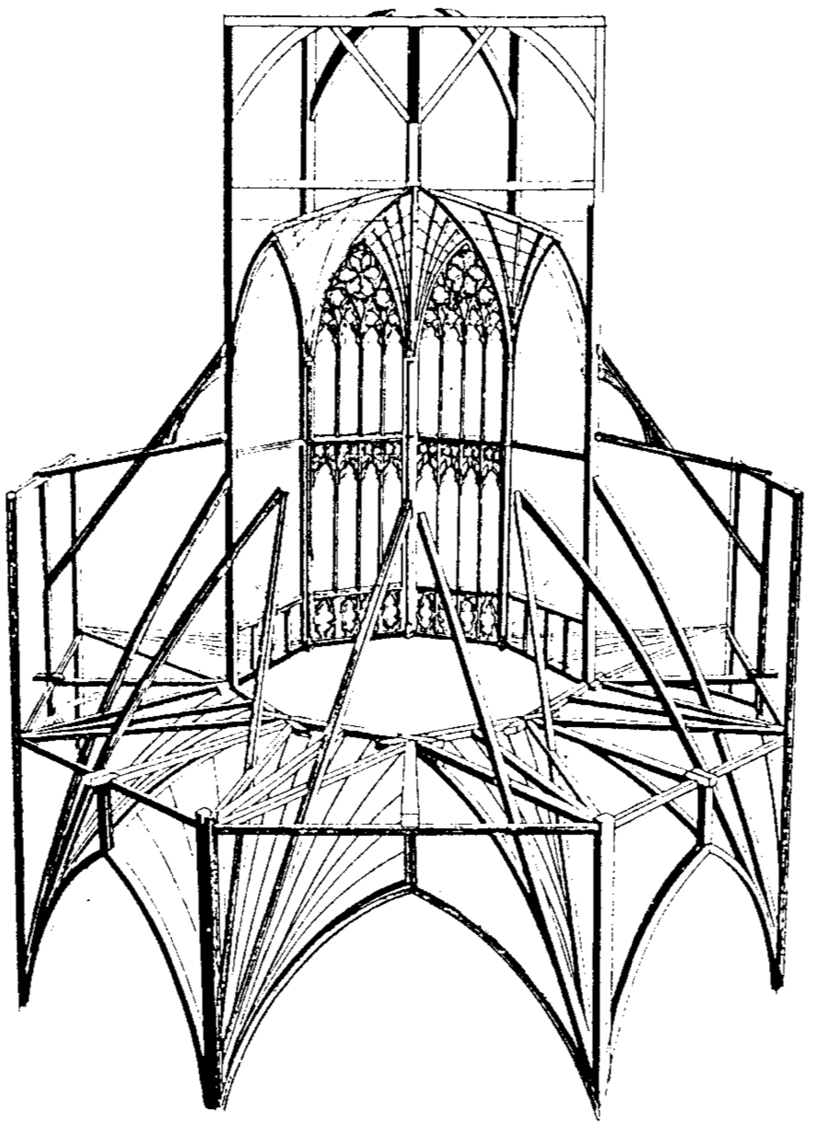

Fig. 16

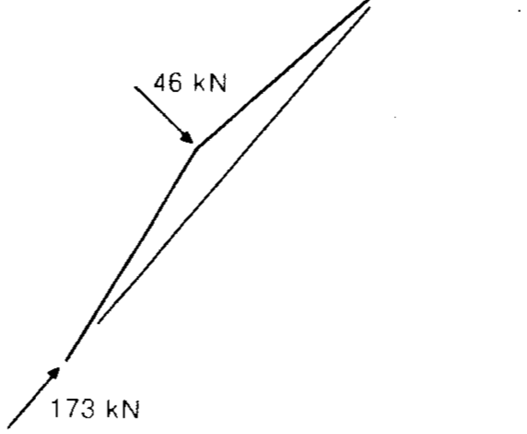

Fig. 17

31. Additionally, external flying buttresses of timber were added to stay further the great posts. These may be seen in Hewitt's conjectural drawing, Fig. 16, and in Bentham's drawing of 1756, Fig. 5, but they are omitted in the cross-section of 1766, Fig. 4. It is not clear from Fig. 5 how the flying buttresses were related to the timber framework of the vault; in any case, Essex removed the buttresses in 1757–62.

32. The 16 diagonal stays prove, on examination, to be in contact with the curved ribs, as sketched in the Walsingham view in Fig. 13. A better model of the action of the forces in a rib might then be thought to be that illustrated in Fig. 17 (cf. Fig. 15). The 'lines of thrust' are now just about containable within the depth (320 mm) of the curved rib and will give rise to little bending. However, the diagonal stay, of section about 300 mm × 300 mm and unsupported length about 11 m is required to provide a lateral force of 46 kN. The stay will itself deflect by over 100 mm under this load and will be stressed in bending to about 30 N/mm^2. It is not surprising that Essex found it necessary to add bracing members in the 18th century (Fig. 13, Essex view).

33. However, the diagonal stays can themselves assist in carrying the weight of the lantern by acting as struts in compression. Indeed, if the weight of 2000 kN is supposed to be taken entirely by these stays, then each pair must contribute an inclined force of about 305 kN to support the load of 250 kN in each of the eight vertical posts. (The Euler buckling load of a single 11 m timber of section 300 mm × 300 mm is about 550 kN.)

34. It will be seen from Fig. 13 (Walsingham view) that the diagonal stays are framed into the posts about half-way between the upper and lower sills (the framing in actuality shows some variation). Thus each post, subjected to a lateral load from the pair of stays of about 175 kN, will be put into bending. The posts may be approximated by a rectangular section 560 mm × 330 mm, with a section modulus of about 10×10^6 mm^3. A simple $Wl/4$ calculation for a load of 175 kN and a span of 6 m leads to a bending stress of 26 N/mm^2, which is again unacceptably high as a secular value.

35. Thus the basic model of Fig. 11 leads to a set of equilibrium forces which can perhaps be generated precariously by reason of the many extra supporting members in the real framework and which are satisfactory except for the main curved supporting ribs, Fig. 15. The ribs must be backed by the diagonal stays, but these in turn then become overstressed. Alternatively, the model in which all the load is taken by the diagonal stays requires excessive bending to be developed in the eight great posts. In practice it may be imagined that the forces from each of these two basic models will combine to support the weight of the lantern. Even so, the stresses will not be at that comfortably low level at which maintenance problems will be slow to arise.

Essex's repairs

36. Indeed, slight repairs are mentioned in the records as having been made during the reigns of Henry VI, Edward IV, Henry VII and Henry VIII. A full report on the octagon was made by Essex in 1757, in which he found the whole structure to be much decayed and improperly repaired and patched. The upper parts were in a ruinous state, and he took the opportunity to remodel the external aspect of the octagon, at the same time removing (as has been mentioned) the external flying buttresses to the lantern.

37. Moreover, Essex added large quantities of timber to the lower parts of the structure, as may be seen in Fig. 13, Essex view (cf. Bentham, Fig. 4). He added stays at a higher level than the original, in fact at the level of the upper sill, together with horizontal and vertical members to help to brace these stays, and he backed the original stays at their weak points where they support the curved ribs.

38. Essex also removed at this time the vertical wall posts, AB in Fig. 8, which were rotten and structurally unnecessary, together with the horizontal straining members between joints A. Thus both of the main load paths for the dead weight of the structure were reinforced by Essex; the main curved ribs could be better relied on, and at the same time a new diagonal stay system was provided.

39. As will be seen from the two Bentham views, Figs 4 and 5, Essex added stone spirelets to the outer corners of the octagon, and he also crowned the eight great posts with pinnacles.

Scott's restoration

40. George Gilbert Scott reported on the lantern of Ely Cathedral in 1862–3,[11] and he declared his object of undoing Essex's work and restoring the lantern as far

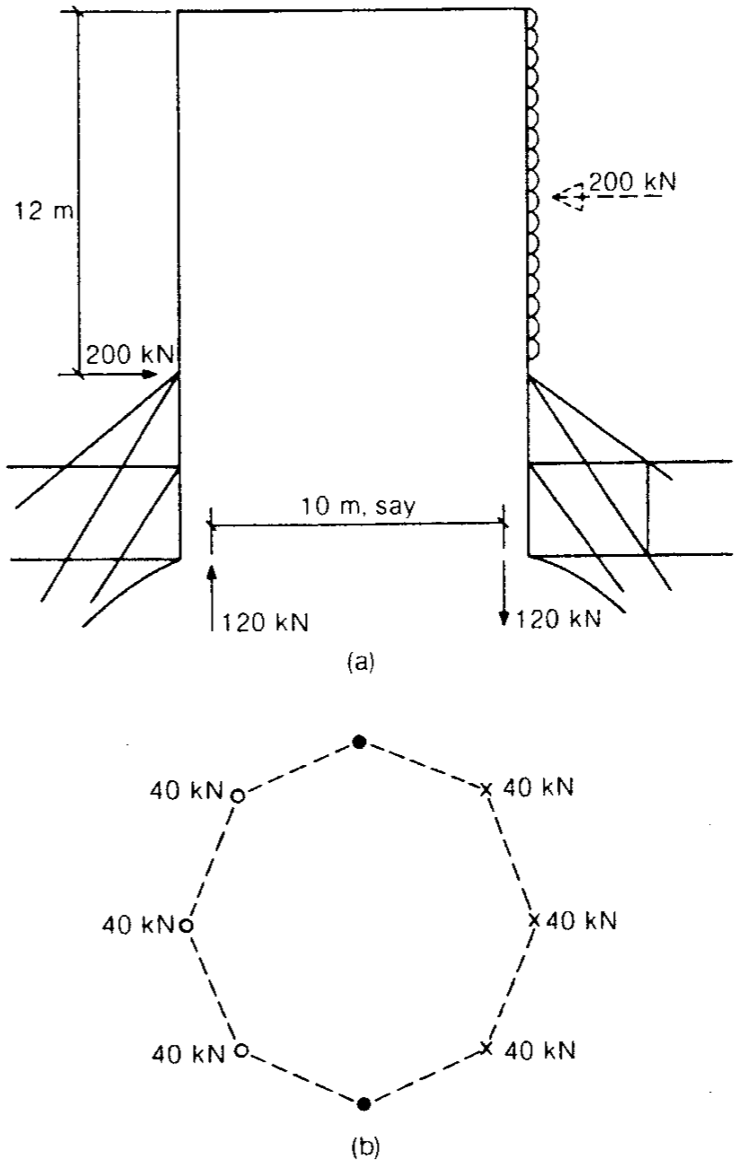

Fig. 18

as possible to its original design. There seems to be no question of the work having been inspired by necessary structural strengthening. Scott had in mind the external appearance of the whole, and, for example, he removed the pinnacles from the eight great posts, while retaining the pinnacles at a lower level. Indeed, he inserted masonry at the centre of the long sides of the supporting octagon, so that the restored flying buttresses could bear against solid work. (The outer octagon thus has 12 spirelets at the present day.)

41. The external buttresses, together with the diagonal stays at a lower level, will help to brace the lantern against wind. An effective wind speed of 48 m/s leads to a total lateral load on the projecting portion of the lantern (about 12 m) of about 200 kN. This load could be carried as shown in Fig. 18, where the flying buttresses have been removed (Essex), and where the reacting moment is assumed to be supplied by additional vertical forces in the eight great posts. The total force required from each group of three posts is about 120 kN, or say 40 kN in each post. Strangely, such an unequal distribution of forces as that shown in Fig. 18(b) will not upset the precarious equilibrium of the basic space frame, as may be appreciated from equation (7).

42. The diagonal stays must between them absorb the horizontal reaction of 200 kN. Scott's restored flying buttresses will take a large part of this load, and a force of the order of 100 kN can easily be supported by these buttresses.

Conclusion

43. The axial forces involved in members of the octagon have magnitudes like 300 kN. A force of 300 kN acting endways on a timber member 300 mm × 300 mm will produce a compressive stress of 3 N/mm^2—a comfortably low value, corresponding to a strain of say $0{\cdot}3 \times 10^{-3}$ N/mm^2. Thus if the timber member has length 10 m, it will shorten by about 3 mm under this load.

44. If, however, the load of 300 kN acts transversely on the same baulk of timber, spanning 10 m and simply supported, then bending stresses of 170 N/mm^2 are generated, and the lateral deflexion is 900 mm. A timber framework, to be effective and to be built with some hope of lasting through the centuries, must be designed so that its members are loaded axially, with bending obviated as much as possible.

45. The timber structure at Ely is not good in this respect. It was singularly unfortunate that a curious basic structure, simultaneously statically indeterminate and a mechanism, was formed by the main framing. Apart from this, a major error was to rely on curved ribs to support the lower sill on which the great posts of the lantern were erected. Architecturally, the use of the curved ribs leads to fine timber vaulting, clearly related to the adjacent stone vault of the choir. Whereas, however, the curved arch style is suitable for relatively heavy vaults made of masonry, it is statically inappropriate for the lighter timber vault. Or rather, it would not have been of importance if the timber vault, like the stone choir vault, was carrying only its own weight. However, the timber vault in the octagon is required to carry the 2000 kN of the lantern, and straight props rather than curved ribs would have been two orders of magnitude more effective structurally. (Aesthetically, of course, straight ribs could well have been a disaster.) As it is, the curved ribs are in principle subjected to high bending stresses insofar as they come into play to support the weight of the lantern.

46. Walsingham (Hurley) provided the diagonal stays, Fig. 13, which will also play their part in carrying the weight of the lantern. A further error in the original

design was to frame these stays into an unsupported section of the great posts, between the upper and the lower sill. To the extent that the stays do help to support the lantern, unacceptably large bending stresses are induced in the great posts.

47. These design errors will have contributed to the ruinous state found by Essex 400 years after the lantern was built. Essex dealt as well as he could with the weakness of the curved ribs by providing back-up framing, and he installed proper diagonal bracing which acts at upper sill level to stabilize the structure. All this was, and is, effective; the fact that he removed the external flying buttresses, and redesigned the exterior of the octagon in a way unacceptable to Scott, does not detract from the fact that we probably owe the continued existence of the octagon to Essex.

48. Scott (with his engineer, R. Reynolds Rowe) thoroughly refurbished both the original main timbers and Essex's additions. Further, he restored the flying buttresses that Essex had removed. Walsingham's design was an architectural masterpiece, but it was, in its original form, something of a structural mistake. Between them, Essex and Scott have provided, after several centuries, a satisfactory structure for the octagon.

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